

# Endpoint regularity of maximal operators in higher dimensions

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We prove the endpoint regularity bound

$$\|\nabla Mf\|_{L^1(\mathbb{R}^n)} \leq C_n \cdot \|\nabla f\|_{L^1(\mathbb{R}^n)}$$

for some maximal functions  $Mf$  in any dimension  $n \geq 1$ . This bound and various related questions have been intensively studied for all sorts of maximal operators. Nevertheless, it has essentially only been resolved on  $L^p$  for  $p > 1$  and in one dimension. We prove it for the uncentered Hardy-Littlewood maximal function of characteristic functions, for general dyadic maximal functions and for the cube maximal function.

The key arguments of the proofs are of geometric nature. For example new variants of the isoperimetric inequality and of the Vitali Covering Lemma are proven and used. All proofs are mostly elementary up to applications of classical results like the relative isoperimetric inequality and the coarea formula.